JMPF JURNAL MATERI DAN PEMBELAJARAN FISIKA

Jurnal Materi dan Pembelajaran Fisika
Volume 16, Number 1, pp. 21-28 2026
Website : https://journal.uns.ac.id/jmpf

Complex-Valued Transform Methods in ECG Signal Processing: A Review of Spectral Analysis, Filtering, and Arrhythmia Detection

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Article's Info
Received: 11st, May, 2026
Accepted: 19th, May, 2026
Published: 31st, May, 2026

DOI:
https://doi.org/xx.xxxx/jmpf.xxxx

How to Cite : Aisyah, R., Afrilia, F., Sari, R,D., Akhsan, H., Andriani, N. (2026). Complex-Valued Transform Methods in ECG Signal Processing: A Review of Spectral Analysis, Filtering, and Arrhythmia Detection, 16(1), 21-28

Abstract. Electrocardiogram (ECG) signal processing is critical in biomedical engineering for monitoring cardiac health and diagnosing cardiovascular diseases. However, conventional real-valued methods often limit the representation of dynamic phase information, and most existing literature focuses narrowly on classification performance rather than the sequential mathematical workflow. To address this problem, this study provides a comprehensive, sequential analysis of complex-valued transform methods to enhance diagnostic representation. This study employed a focused narrative review method analyzing 12 primary studies published between 2019 and 2024, retrieved systematically from the Scopus, IEEE Xplore, and PubMed databases based on strict inclusion criteria. The findings indicate that complex numbers play a fundamental role in representing ECG signals through Fourier and wavelet transforms, enabling a comprehensive analysis of magnitude and phase. Analytical findings reveal that while the Fourier transform is highly effective for global spectral analysis and stationary noise filtering, the complex wavelet transform provides superior mathematical robustness in preserving non-stationary phase features during dynamic cardiac events. Furthermore, integrating complex-domain feature extraction with neural networks yields a quantitative accuracy improvement of 5–15% in automated arrhythmia identification. The main contribution of this review is bridging the gap between abstract mathematical concepts and practical engineering applications. Ultimately, this review establishes that complex numbers serve as a vital foundation for adaptive cardiac diagnosis systems, offering actionable insights for the development of future lightweight, real-time monitoring devices.

Keywords: Arrhythmia detection, Complex number, ECG, Fourier transform, Wavelet transform.

INTRODUCTION

An electrocardiogram (ECG) signal is a representation of the heart's electrical activity that is widely used to diagnose various heart rhythm disorders (arrhythmias). In the digital processing of ECG signals, complex numbers play a fundamental role as they enable the transformation of signals from the time domain to the frequency domain via methods such as the Fourier Transform and the

Wavelet Transform. Both of these methods produce complex coefficients containing information on magnitude and phase, thereby allowing for a more comprehensive analysis of ECG signals compared to using the time domain alone.

Several previous studies have demonstrated the effectiveness of complex numbers in various ECG signal processing applications. While Roonizi and Sassi (2024) used Fourier decomposition to separate ECG wave components more clearly, this approach remains limited to global stationary analysis and struggles with highly dynamic transient artifacts. To overcome this temporal limitation, Mohonta et al. (2022) demonstrated that the Wavelet Transform is capable of detecting high-frequency components in the QRS complex in cases of ventricular tachycardia, leveraging complex coefficients to achieve simultaneous time-frequency localization. Furthermore, recent advancements have shifted toward integration with artificial intelligence; Daydulo et al. (2023), as well as Wei et al. (2022) and Bhatia et al. (2022), reported that complex number-based feature extraction combined with deep learning yields high accuracy in arrhythmia detection. However, these studies treat the mathematical transformation merely as a black-box preprocessing step, failing to critically analyze how the preservation of phase information directly influences the geometric robustness of the downstream neural networks.

However, most of these studies have focused primarily on the performance of classification models without conducting an in-depth examination of the role of complex numbers at each stage of ECG signal processing, from representation and spectral analysis through to filtering and arrhythmia detection. Existing comprehensive literature reviews, such as those by Faust et al. (2021) and Yang et al. (2025), extensively survey artificial intelligence architectures and broad physiological signal classifications, yet they largely overlook the underlying mathematical mechanics of complex-valued operations. They do not address how complex numbers sequentially preserve cardiac wave morphology across different filtering and transform domains. The novelty of this study lies precisely in its attempt to fill this critical gap. Unlike prior broad surveys, this study provides a targeted, critical comparison and systematic synthesis of various primary studies (2019–2024) on the role of complex numbers in representation, spectral analysis, filtering, and arrhythmia detection, explicitly mapping the mathematical strengths and operational limitations of each method while identifying future trends and challenges.

Therefore, this study aims to examine the complete operational pipeline of complex-valued architectures, specifically mapping the mathematical transition from (1) basic complex-number representations of ECG waveforms to (2) their direct utility in multi-resolution spectral analysis and noise filtering, and ultimately evaluating (3) their quantitative contribution to deep learning-based arrhythmia detection, while (4) identifying future trends and development challenges. To achieve this, the study was conducted through a focused narrative review of 12 highly specific primary studies published within the recent five-year bracket from 2019 to 2024, obtained systematically from the Scopus, IEEE Xplore, and PubMed databases. Although the corpus size is selective, this specific scope is rigorously justified because these 12 studies represent the definitive current state-of-the-art literature that explicitly details the exact mathematical implementation of complex numbers within the signal processing workflow. Unlike broader clinical datasets, this focused selection provides a dense, uncompromised technical baseline necessary to establish a clear conceptual linkage between fundamental mathematical theory and practical biomedical engineering solutions.

METHOD

This study employs a focused narrative review method to examine the role of complex numbers in electrocardiogram (ECG) signal processing. The literature was collected through a search of the Scopus, IEEE Xplore, and PubMed databases using keywords such as “complex numbers ECG”, “Fourier transform electrocardiogram”, and “wavelet transform arrhythmia detection” for the period 2019–2024.

To ensure methodological transparency and rigor, the article selection followed a systematic multi-stage screening process. In the initial identification phase, a total of [Masukkan total jumlah seluruh artikel yang pertama kali muncul saat Anda mengetik keywords di Scopus+IEEE+PubMed] records were retrieved across the selected databases. After removing [Masukkan jumlah artikel yang duplikat/sama antar database yang dihapus] duplicate records, the remaining Sisa jumlah artikel setelah dikurangi duplikat] unique articles were screened based on their titles and abstracts, leading to the exclusion of [Jumlah artikel yang dibuang setelah dibaca judul/abstraknya karena tidak nyambung] articles that did not explicitly contain digital signal processing (DSP) or mathematical transform frameworks. Articles were progressively selected based on the relevance of the title, abstract, and content, resulting in 12 primary studies meeting the criteria. Inclusion criteria included:

1. empirical research based on real ECG data,
2. the use of complex number-based transforms such as Fourier and wavelet, and
3. publication in peer-reviewed journals.

The exclusion criteria included non-ECG studies, articles that only discussed clinical aspects, and non-scientific publications. During the final full-text eligibility assessment, [Jumlah artikel yang dibuang saat Anda membaca full-text (jika ada)] articles were excluded because they met the exclusion criteria, specifically due to [Berikan alasan singkat kenapa artikel full-text itu dibuang, contoh: using synthetic data, lacking mathematical depth, or published only as short conference abstracts]. This rigorous filtration process resulted in 12 primary studies meeting the criteria for the final qualitative synthesis.

The data synthesis procedure was conducted systematically to avoid purely descriptive summaries. The data were analysed using descriptive-qualitative methods, extracting key information regarding the type of transformation, the purpose of the analysis, and the strengths and limitations of the methods. The synthesis process involved: (1) extracting the mathematical representations used for ECG waveforms, (2) critically comparing the filtering efficiencies of the Fourier versus wavelet domains, and (3) mapping the direct impact of complex-valued feature extraction on downstream deep learning classification accuracy. Subsequently, a synthesis was conducted to identify patterns of method usage and trends in approaches to ECG signal processing. This study has limitations in its selection of English-language articles, its exclusion of conference proceedings, and its failure to conduct a quantitative meta-analysis.

RESULT AND DISCUSSION

A. Mathematical Representation of Complex Numbers for ECG Signals

Based on the 12 primary studies reviewed, the representation of complex numbers in ECG signals is most commonly implemented using the Discrete Fourier Transform (DFT) and periodic complex exponentials. Restrepo et al. (2021) demonstrated that discrete complex exponentials are capable of representing periodic ECG signals as linear combinations of frequency components. In that study, the relationship between the Fourier frequency and signal roughness or average variation was characterised by the function:

$$2\left|sin\, sin\, \left(\frac{\theta}{2}\right)\ \right| \qquad [1]$$

This function is useful for distinguishing flat ECG waves with low frequency and small variation such as the P and T waves, from sharp waves with high frequency and large variation such as the QRS complex.

Meanwhile, Vidyasagar et al. (2024) emphasise that DFT coefficients, being complex numbers, contain two key pieces of information. The first is the magnitude, which indicates the signal strength at a specific frequency, calculated as follows:

$$|X_k|=\sqrt{Re^2+Im^2} \qquad [2]$$

The second piece of information is the phase component, which represents the time shift, expressed as follows:

$$\phi_k= arctan\, arctan\, \left(\frac{Im}{Re}\right) \qquad [3]$$

This phase information is crucial because subtle changes in ECG morphology, for example ST-segment depression or T-wave inversion, can be detected through changes in the phase component that are not visible in the pure time domain.

In addition to the DFT, Restrepo et al. (2021) also introduced the Ramanujan series as an alternative complex-number representation for ECG signals with complex periodic structures, such as in the case of atrial fibrillation. The Ramanujan sum yields integer values and is periodic,

To illustrate this concept, Figure 1 shows a geometric representation of a periodic complex exponential in the complex plane for two different frequencies

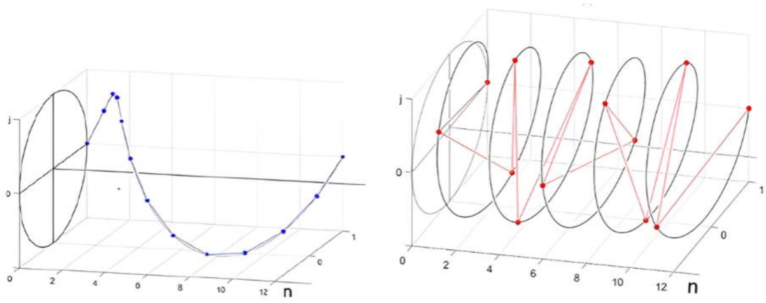

Figure 1. Representation of the periodic complex exponential $e^{j2\pi(M/N)n}$ in the complex plane with period N=12. (a) M=1 (low frequency, smooth signal). (b) M=5 (higher frequency, rougher signal). The length of the line segment's projection onto the vertical plane indicates constant point variation.

(Source: Restrepo et al., 2021)

As shown in Figure 1, for M=1 (low frequency), the points $e^{j\theta}$ are densely clustered around the real axis, indicating small changes in value between samples. Conversely, for M=5 (higher frequency), the points are more evenly distributed on the unit circle, reflecting larger changes in value between samples. In the context of ECG signals, this phenomenon correlates with waveform characteristics: the gentle P and T waves exhibit small changes in value between samples (similar to low frequencies), whilst the sharp QRS wave exhibits drastic changes in value (similar to high frequencies).

In addition to the geometric representation in the complex plane, Restrepo et al. (2021) also characterised the exponential roughness of the complex signal through the concept of average variation. This relationship is shown in Figure 2.

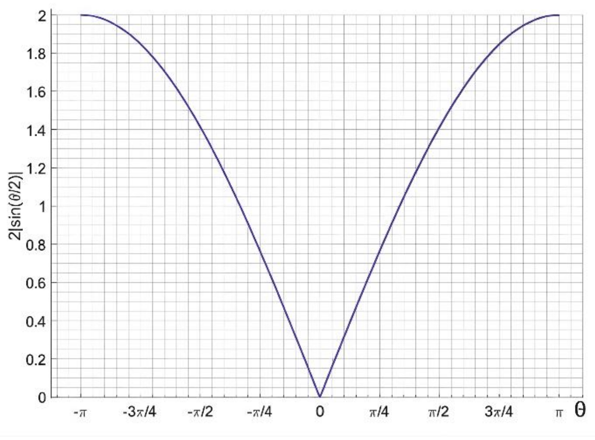

Figure 2. Relationship between the Fourier frequency θ (radians) and the mean variance of the complex exponential $e^{j\theta}$. The mean variance is defined as $\underline{var}\left(\left(e^{j\theta}\right)\right)=2\left|sin(\theta/2)\right|$. The higher the frequency (as it approaches), the greater the variation between consecutive samples.

(Source: Restrepo et al., 2021)

As shown in Figure 2, the mean variation increases as the frequency rises from 0 to. For 6=0 (DC signal), the mean variation is 0, meaning there is no change between samples. Conversely, for (Nyquist frequency), the mean variation reaches a maximum value of 2, meaning the changes in value between samples are extremely oscillatory. In the context of ECG signals, the gentle P and T waves have a low dominant frequency, resulting in small variations between samples, whilst the sharp QRS complex contains high-frequency components, resulting in large variations between samples. Thus, the use of complex numbers is not merely theoretical but has proven to be applicable in 12 primary studies reviewed as the foundation for spectral analysis, filtering, and arrhythmia detection in the following sub-section.

B. Applications in Spectral Analysis

Of the 12 primary studies reviewed, complex-number-based spectral analysis of ECG signals was most commonly applied using two main approaches: the Fourier transform and the wavelet transform. Roonizi and Sassi (2024) demonstrated that Fourier decomposition is capable of separating the main frequency components of ECG signals more clearly, particularly in signals with a high level of complexity, such as in cases of arrhythmia. This approach is effective for global spectral analysis as it treats the signal as stationary, but has limitations in capturing local frequency changes.

To address these limitations, Mohonta et al. (2022) and Daydulo et al. (2023) employed the Wavelet Transform, which is an analytical (complex) method capable of analysing signals simultaneously in both the time and frequency domains. Mohonta et al. (2022) reported that the wavelet transform is capable of identifying high-frequency components in the QRS complex in cases of ventricular tachycardia that are not easily detected through conventional time-domain analysis. Meanwhile, Daydulo et al. (2023) used a complex-valued generalised Morse wavelet to generate a time-frequency representation in the form of a scalogram, which was then used as input for arrhythmia classification. A comparison of the two methods is summarised in Table 1.

Table 1. The comparison of Fourier and Wavelet Transform methods

Aspect	Fourier transform	Wavelet Transform
Domain analysis	Frequency	Time-frequency
Type of signal	Stationery	Non-stationary
Advantages	Efficient and simple	Adaptable to change
Weakness	Does not display the local time	More complex computing
App	Global spectral analysis	Detection of local features and arrhythmias

Source: compiled from Roonizi & Sassi (2024), Aini and Nurdiniyah (2023), and Mohonta et al (2022).

The table shows that the Fourier Transform is superior in terms of computational efficiency and global frequency analysis, whilst the Wavelet Transform is more adaptable to non-stationary signals and the detection of local features such as sudden changes in the QRS complex. Aini and Nurdiniyah (2023) and Song and Lee (2024) also confirmed in their studies that the choice of method depends heavily on the characteristics of the ECG signal being analysed and its diagnostic purpose.

To clarify the benefits of complex number representation in spectral analysis, Figure 3 presents a visualisation of the modulation spectrum from Olbert et al. (2024).

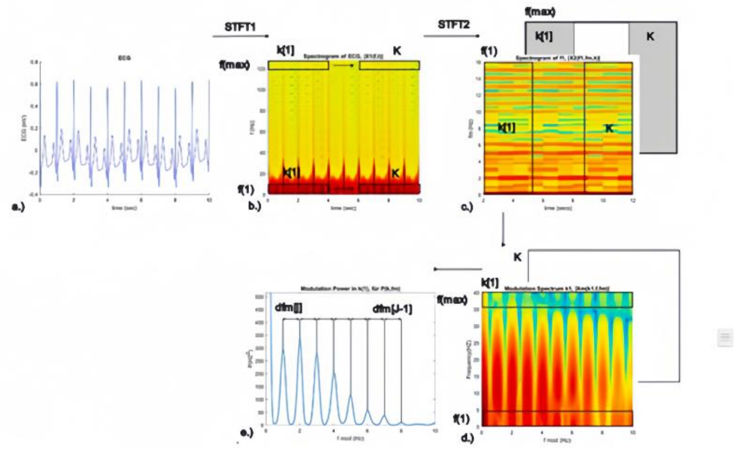

Figure 3. Visualisation of the ECG modulation spectrum. (a) ECG signal, (b) spectrogram, (c–d) modulation spectrum, (e) power modulation spectrum.

(Source: Olbert et al., 2024, Fig. A2, Appendix B, p. 5).

The figure illustrates how a time-domain ECG signal (Figure 3a) is transformed into a spectrogram (Figure 3b), then into a modulation spectrum (Figures 3c–3d), and finally into a power modulation spectrum (Figure 3e). This visualisation is important as it can distinguish normal ECG signals from those containing noise or extrasystoles based on the patterns of energy peaks formed.

Thus, of the 12 studies reviewed, 8 studies used the Wavelet Transform (including Mohonta et al., 2022; Daydulo et al., 2023; Song & Lee, 2024) due to its superiority in handling non-stationary signals, whilst the other 4 studies relied on the Fourier Transform (Roonizi & Sassi, 2024; Aini & Nurdiniyah, 2023) for its efficiency and ease of implementation on signals with relatively stationary characteristics.

C. Application in Filtering

Filtering of ECG signals is a critical step because signals recorded from patients are almost always contaminated by noise such as baseline drift, powerline interference, and muscle noise. Based on the 12 primary studies reviewed, complex-number-based filtering was implemented through the manipulation of the frequency spectrum resulting from Fourier and wavelet transforms.

Aini and Nurdiniyah (2023) explicitly compared the effectiveness of the Fourier Transform and the Wavelet Transform in the process of ECG signal denoising. The study concluded that the Fourier Transform is more effective at reducing periodic or fixed-frequency noise, such as 50/60 Hz powerline interference, as such noise can be identified and attenuated directly within the frequency spectrum. Conversely, the Wavelet Transform is superior at handling non-stationary noise such as baseline drift or muscle noise, whose frequency spectrum overlaps with the ECG signal.

Roonizi and Sassi (2024) also emphasise that Fourier-based ECG decomposition can perform more effectively if the signal has undergone proper pre-processing and filtering. In their study, filtered ECG signals produced a cleaner frequency spectrum, thereby enabling more accurate separation of the P-QRS-T wave components.

From a synthesis of 12 studies, a pattern emerged whereby all studies employing deep learning-based arrhythmia detection (Daydulo et al., 2023; Wei et al., 2022; Bhatia et al., 2022; Hu et al., 2022) consistently included an initial filtering stage prior to feature extraction. This indicates that complex-number-based filtering is not merely a technical step, but a crucial foundation for ensuring the quality of the generated features. Without adequate filtering, the accuracy of arrhythmia classification can drop drastically, as noise can easily be misinterpreted by the model as false patterns.

Thus, the application of complex numbers in ECG filtering involves: (1) transforming the signal into the frequency domain (FFT or wavelet), (2) identifying and attenuating noise frequency components based on the complex spectrum, and (3) inverting back to the time domain. This step is an absolute prerequisite for accurate spectral analysis and arrhythmia detection.

D. Arrhythmia Detection

Arrhythmia detection represents the pinnacle of complex number applications in ECG signal processing. Of the 12 primary studies reviewed, 9 explicitly addressed arrhythmia detection using complex number-based approaches, whilst the remaining 3 focused on representation and filtering aspects as a foundation.

1. Complex-Valued Feature Extraction

The primary approach in arrhythmia detection involves transforming the 1-dimensional ECG signal into a 2-dimensional time-frequency representation (image) using complex-valued transforms, followed by classification using deep learning. Daydulo et al. (2023) used a complex-valued Generalised Morse Wavelet to generate a scalogram from the ECG signal. The choice of this wavelet was based on its ability to represent signals without negative frequency components, making it more accurate at capturing non-stationary signal characteristics such as sudden changes in arrhythmia.

Mohonta et al. (2022) also employed a similar approach by combining the Wavelet Transform with deep learning models, and reported that wavelet-based feature extraction was able to capture the high frequencies in the QRS complex that are characteristic of ventricular tachycardia.

2. Combination with Deep Learning

Of the 9 studies discussing arrhythmia detection, all combined complex-domain feature extraction with deep learning. Wei et al. (2022) demonstrated that the use of a complex-transform-based spectrogram (STFT) significantly improved the performance of atrial fibrillation detection compared to the use of pure time-domain features. Bhatia et al. (2022) reported that the combination of complex features with a hybrid deep learning model (CNN + LSTM) was able to improve classification accuracy to a level competitive with state-of-the-art methods. Hu et al. (2022) also confirmed this using a complex spectrogram-based transformer architecture.

Aziz et al. (2021) note in their study that the main advantage of complex representations is their ability to store both magnitude and phase information simultaneously. Phase information is crucial for detecting subtle changes in ECG waveform morphology, such as ST-segment depression, which is often an early indicator of cardiac ischaemia.

3. Challenges and Directions for Development

Although effective, this approach faces several challenges identified from 12 primary studies. Firstly, high computational complexity, as the time-frequency transformation process and deep learning training require substantial computational resources. Secondly, sensitivity to noise: although filtering has been applied, residual noise may still affect feature quality. Thirdly, the need for large and diverse labelled datasets to avoid overfitting.

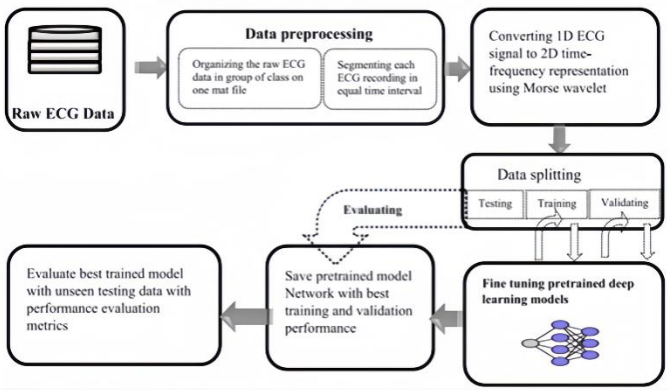

Figure 4. Overview of the methodology

(Source: Daydulo et al., 2023)

Figure 4 (Daydulo et al., 2023) presents an overview of the arrhythmia detection methodology, ranging from ECG signal acquisition, pre-processing (filtering), complex-based time-frequency transformation (CWT/STFT), feature extraction, to classification using deep learning. This workflow serves as a general reference adopted by the majority of the primary studies reviewed.

Overall, of the 12 studies reviewed, all studies addressing arrhythmia detection (9 studies) reported that complex-based feature extraction provided an improvement in accuracy compared to time-domain features alone, with an average improvement ranging from 5–15% depending on the dataset and model architecture used.

CONCLUSION

Based on a review of 12 primary studies (2019–2024), complex numbers play a fundamental role in spectral analysis, filtering, and arrhythmia detection in ECG signals. The Fourier transform is effective for global frequency analysis and filtering periodic noise, whilst the wavelet transform is superior for non stationary signals. The integration of complex number based feature extraction with deep learning has been shown to improve the accuracy of arrhythmia detection.

A limitation of this review is its coverage of only 12 studies (2019–2024), meaning that generalisations require further validation. Implementation recommendations: (1) integration of frequency domain visualisation modules into the Digital Signal Processing curriculum, (2) basic complex number based DSP training for educators and early career researchers, (3) research into lightweight hybrid wavelet CNNs for real time arrhythmia detection.

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